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COMPARISON OF MONTE-CARLO ION CYCLOTRON HEATING MODEL WITH FULL-WAVE LINEAR ABSORPTION MODEL

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ABSTRACT

To fully account for the wave-particle interaction physics in ion-cyclotron resonant frequency (ICRF) heating experiment, finite orbit effects and non-Maxwellian distribution have to be self-consistently coupled with full-wave solutions. For this purpose, the 5-D Monte-Carlo code ORBIT-RF is being coupled with the 2-D full wave code AORSA to iteratively evolve the ion distribution in 4D spatial-velocity-space that is used to update the dielectric tensor in AORSA for evaluating the full-wave fields. In this paper, it is demonstrated that using the full-wave fields from a Maxwellian dielectric tensor in AORSA and confining the resonant ions to their initial orbits in ORBIT-RF, ORBIT-RF largely reproduces the AORSA linear wave absorption profiles for fundamental and higher harmonic ICRF heating. An exception is an observed inward shift of the ORBIT-RF absorption peak for high harmonics near the magnetic-axis compared with that of AORSA, which can be attributed to a finite orbit width effect. The success of this verification supports the validity of the Monte-Carlo wave-particle interaction model and the readiness of the iterative coupling between ORBIT-RF and AORSA for an improved modeling of ICRF heating experiments.
I. INTRODUCTION

Plasma heating process using ion-cyclotron resonant frequency (ICRF) wave in tokamaks generates a large population of energetic ions in plasma distribution function. Finite drift motion of these non-thermal ions along the torus may significantly modify the ICRF wave propagation and absorption in the plasma. More interestingly, neutral beam injection or fusion reaction produces a significant concentration of energetic beam or alpha particles even before the ICRF wave heating power is turned on. Therefore, theoretical modeling of ICRF wave — plasma interaction at ion cyclotron harmonics requires that plasma distribution solver should not only take into account finite drift orbit effect, but also be coupled with wave field solver in a self-consistent way.

As previous work, the 2-D global-wave field solver AORSA\(^1\) is combined with the 3-D bounce-averaged Fokker-Planck (F-P) plasma distribution solver CQL3D.\(^2\) AORSA, including up-shift effect in parallel wave number with no limitation to high ion cyclotron harmonics heating, solves the integral form of wave equations to all orders in Larmor radius and calculates wave electric fields. Bounce-averaged diffusion coefficients, derived directly from AORSA wave fields, are fed to CQL3D to calculate a long-time response of plasma distribution function. However, since in CQL3D, the plasma distribution function is computed using zero orbit width approximation, AORSA/CQL3D does not account for both finite orbit drift of RF-induced non-thermal species on ICRF wave power absorption and their radial diffusion across flux surfaces. Therefore, the AORSA/CQL3D, while a significant step beyond the linear absorption model, may not provide a complete description of the ICRF wave – non-thermal plasma interaction simulations.

For example, quantitative agreement is found between AORSA/CQL3D in the ICRF thermal minority ion heating experiment at fundamental harmonic in the Alcator C-Mod tokamak.\(^3\) However, the application of AORSA/CQL3D to neutral beam preheated plasmas in the DIII-D fast wave current drive (FWCD) experiments\(^4\) predicted a strong power absorption on injected deuterium beam ions, while experimental observation from a neutron diagnosis\(^5\) indicated absorption at a much lower level. Ion cyclotron resonance layers and strong quasi-linear (Q-L) diffusion take place on the surfaces of constant magnetic field, roughly vertical planes. Since energetic ions are known to exhibit large banana orbits that deviate significantly from flux surfaces, their orbit topology can move them away from (or towards) vertical resonance layers. This effect, currently not modeled in AORSA/CQL3D, is expected quantitatively to modify the ICRF wave propagation and absorption in non-thermal plasmas. Therefore, the full wave approach combined with the...
F-P distribution solver with zero-orbit width might produce inaccuracy specifically in the interpretation of DIII-D high harmonic ICRF wave heating experiment in beam preheated plasma in which large orbit energetic ions are expected to contribute significantly to wave absorption.

As alternative numerical approach, the 2-D linear full wave solver TASK/WM\textsuperscript{6} is being coupled to the 5-D drift kinetic equation solver GNET.\textsuperscript{7} Hellsten, et al. also developed a SELFO\textsuperscript{8} code where the FIDO,\textsuperscript{9} solving orbit-averaged differential equations for invariants of motion of unperturbed orbits in (E, Λ, P₀) space, is coupled with the 2-D global wave code LION.\textsuperscript{10} Here, E is the particle energy, Λ = μB₀/E, μ the magnetic moment, P₀ the canonical angular momentum. Unique features of GNET and FIDO compared with CQL3D are that they take into account finite orbit width effect of non-thermal particles in the plasma distribution. However, these simulation packages still have some limitations when applied to high harmonic ICRF heating experiments in shaped plasmas, since TASK/WM is currently valid up to second harmonic, and FIDO is developed for a circular shaped plasma.

To remove the limitations in these approaches, the code ORBIT-RF,\textsuperscript{11} as a self-consistent component for coupling to a full-wave code, is developed with features to take into account finite drift orbit effect, orbit dependent wave-particle interaction time calculation and no limitations to higher harmonic heating in arbitrary 2D axisymmetric numerical magnetic equilibrium. ORBIT-RF follows trajectories of test ions by solving a set of Hamiltonian ion drift orbit equations\textsuperscript{12} in guiding center variable coordinate (X\textsubscript{G}, ρ\textsubscript{∥}, μ, θ) under Coulomb collisions and Q-L diffusive heating. Here, X\textsubscript{G} is the guiding center, ρ\textsubscript{∥} the parallel gyroradius, defined by ρ\textsubscript{∥} = v\textsubscript{∥}/Ω, Ω the ion cyclotron frequency, v\textsubscript{∥} the parallel velocity, μ the magnetic moment, and θ the poloidal angle. These equations of motions are directly solved using a Runge-Kutta fourth order integration scheme in every time step (∆t is usually \( \sim 10^{-2} \) of transit time). Change in pitch angle of test ions due to Coulomb collisions is calculated using Monte-Carlo operators. The Q-L heating model implemented in ORBIT-RF is described in detail in Sec. II.

To couple AORSA and ORBIT-RF self-consistently, wave field amplitude and its spatial pattern computed from AORSA are fed to ORBIT-RF through the transformation of coordinates from cylindrical to Boozer.\textsuperscript{13} In turn, ORBIT-RF provides information on evolved ion distribution function back to AORSA. Since the distribution from ORBIT-RF might be noisy due to Monte-Carlo technique, development is needed to construct a differentiable bounce-averaged distribution function from particle-based distribution function for feedback to AORSA. This work is still under progress through collaboration with the RF SciDAC\textsuperscript{14} team. This paper presents results of a systematic comparison between ORBIT-RF wave absorption predictions using the same full-wave field from AORSA and those directly predicted from AORSA Maxwellian dielectric tensor. Results
demonstrate that using the full-wave fields from a Maxwellian dielectric tensor in AORSA and confining the resonant ions to their initial orbits in ORBIT-RF, ORBIT-RF largely reproduces the AORSA linear wave absorption profiles for fundamental and higher harmonic ICRF heating. An exception is an observed inward shift of the ORBIT-RF absorption peak for high harmonics near the magnetic-axis compared with that of AORSA, which can be attributed to a finite orbit width effect. Self-consistent iterative results will be left for a future paper.

This paper is organized as follows. In Sec. II, theoretical models implemented in ORBIT-RF, which is based on a Monte-Carlo particle Q-L heating approach, and AORSA, which is based on a continuum dielectric tensor approach, for computation of ICRF wave power absorption in the plasma, are described. To crosscheck these absorption models, a “model problem” is set-up by simplifying initial plasma and wave conditions, which is described in Sec. III.A. This “model problem” is applied to the ICRF wave fundamental to high harmonic heating experimental conditions in the Alcator C-Mod and DIII-D tokamaks. Results of extensive benchmarking activities involving the RF SciDAC group are presented and discussed in Sec. III.B. They indicate that finite orbit effect of energetic trapped particles becomes significant at high harmonics when non-thermal ions interaction with waves become important. Lastly, a summary is given in Sec. IV.
II. ICRF WAVE POWER ABSORBED BY THE PLASMA

The standard Q-L diffusive heating theory\textsuperscript{15} assumes that relative phase difference between wave and particle is strongly decorrelated through successive cyclotron resonance encounters, leading to orbit stochasticity, while the “kicks” at each encounter remain very small since it is an expansion with the wave field as a small parameter up to second order. It has been widely used for numerical modeling of ICRF wave plasma resonant interaction to not only interpret present-day ICRF wave heating experiments in conventional tokamaks, and but also predict future burning plasma experiments in ITER\textsuperscript{16}.

ORBIT-RF uses a Q-L model, however relies on some other sources to provide the wave electric fields. AORSA\textsuperscript{1} solves the wave fields from Maxwell’s equations, with the oscillating current related to the wave fields by a constitutive relation (conductivity tensor). This tensor requires knowledge of the plasma distribution function. AORSA can choose the particle distributions to be Maxwellians in which case the conductivity tensor can be analytically obtained and linear power absorption is reproduced. AORSA can also accept particle distribution information from another sources. For example, CQL3D provides the particle distribution in Q-L approximation with zero-orbit width, while ORBIT-RF does the particle distribution also in Q-L approximation but with the added realism of finite orbits and cross-flux collisional diffusion. Thus ORBIT-RF and AORSA are mutually interdependent, which motivates the present effort to couple the two codes to achieve a predictive model for ICRF heating experiments. A first step in the coupling is to demonstrate that OBIT-RF, using the wave fields calculated from AORSA assuming Maxwellian distributions, can reproduce the linear absorption features computed from AORSA for a range of cyclotron harmonics. This constitutes the thrust of the present study.

In ORBIT-RF, locally absorbed power is calculated, as expressed in (1), by summing perpendicular “kicks” of each particle when particles pass through its resonance locations,

\[
P_{RF} = \sum_{\text{particles}} \left[ \langle \Delta \mu \rangle + R_s \sqrt{\langle \Delta \mu^2 \rangle} \right].
\]  

(1)

Since the Q-L theory is based on stochastic diffusion of plasma distribution function in velocity space, a random walk model is introduced, as shown inside a bracket in Eq. (1), to reproduce stochastic nature in \( \mu \) space where \( \mu \) is the magnetic moment and \( R_s \) a
random number satisfying Gaussian probability distribution. A time independent mean change, \( \langle \Delta \mu \rangle \), representing drag, and rapidly fluctuating part \( \sqrt{\langle \Delta \mu^2 \rangle} \), representing dispersion, are derived by connecting Brownian motion theory of individual particle to Fokker-Planck equation by Chandrasekhar.\(^{17}\) As a result, \( \langle \Delta \mu \rangle \) is formulated as\(^{11}\)

\[
\langle \Delta \mu \rangle = \frac{\pi q^2 l^2 \Omega^2}{m \omega^2 B} K[E_+]^2 \times \left[ J_{l-1} + e^{2i\theta_k} \frac{E_-}{E_+} J_{l+1} \right]^2 \\
+ \mu \left[ 2 \left( J_{l-1} + e^{2i\theta_k} \frac{E_-}{E_+} J_{l+1} \right) \left( \frac{\partial J_{l-1}}{\partial \mu} + e^{2i\theta_k} \frac{E_-}{E_+} \frac{\partial J_{l+1}}{\partial \mu} \right) \right] \delta(w_l),
\]

where \( J_{l \pm 1} \) is the \((\pm 1)\)th order Bessel function of the first kind, \( l \) the ion cyclotron harmonic number, \( w_l = \omega - l\Omega - k|v||\), determining the resonance condition, \( \omega \) the wave frequency, \( \Omega \) the ion cyclotron frequency, \( k \) the parallel wave number, \( \Omega || \) the parallel velocity, \( \rho = v_\perp / \Omega = \sqrt{2mB/\Omega} \), \( B \) the magnetic field, \( v_\perp \) the perpendicular velocity, \( k_\perp \) the perpendicular wave number, \( m \) the test ion mass, \( E_+ \) and \( E_- \), defined as \( E_\pm = (E_x \pm iE_y) \), the left-hand and right-hand polarized components of wave electric field, \( \theta_k \), defined as the direction of wave in \( x-y \) plane, \( \cos \theta_k(x,y) = k_x/k_\perp \), \( \sin \theta_k(x,z) = k_y/k_\perp \). A factor \( K \), associated with the integral over \( v_\parallel \), physically relevant to a resonant interaction time, is inserted to account for correlation effect when energetic particle orbit intersects two resonances close to each other or resonance is located at turning point where \( \Omega = 0 \). Generalized expressions for \( K \) with a finite \( k || \), defined as \( K = \left[ \left( |\hat{\omega}_l|/2\pi \right) |f_1^{(0)}| \right]^2 \), are expressed as,

\[
I_l^{(0)} = \begin{cases} 
\frac{2\pi}{|\hat{\omega}_l|_{res} + k||v||_{res}^{(0)}} & \text{if } |\hat{\omega}_l|_{res} + k||v||_{res}^{(0)} \geq \frac{1}{2} |\hat{\omega}_l|_{res} + k||v||_{res}^{(0)} \tau \\
\frac{2\pi A_l}{|\hat{\omega}_l|_{res} + k||v||_{res}^{(0)}}^{1/3} & \text{if } |\hat{\omega}_l|_{res} + k||v||_{res}^{(0)} \geq \frac{1}{2} |\hat{\omega}_l|_{res} + k||v||_{res}^{(0)} \tau
\end{cases}
\]

The expression (3) is one of unique features in ORBIT-RF since the second derivative term in the denominator in (3) is not currently taken into account in AORSA or CQL3D. The expression for \( \langle \Delta \mu^2 \rangle \) is similarly obtained in a straightforward way.\(^{11}\)

In AORSA, local absorbed ICRF wave power is calculated in two ways.\(^{4}\) First, \( P_{RF} \) is calculated locally with wave electric fields, as following,
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\[ RF = \frac{\pi \varepsilon_0 \omega_p^2}{2 \omega \mu} \text{Re} \left( \int_{v_{\perp}}^{\infty} \int_{v_{\parallel}}^{\infty} \left[ \sum_{k_2} \sum_{k_1} \tilde{E}_{k_{\perp}}^{(i)} \frac{a_{l_{\perp}}^{(2)}}{a_{l_{\parallel}}^{(1)}} \right] \right) \), \quad (4)

where the factor \( U \) contains velocity space derivatives of the normalized distribution function, \( \tilde{E}_{k_{\perp}}^{(i)} \) the electric field vector, defined by \( \tilde{E}_{k_{\perp}}^{(i)} = C(\beta) \tilde{E}_{\|} e^{i(k_{\perp} + \beta)} \), \( C(\beta) \) the rotation matrix, \( \omega_p \) the plasma frequency, and \( a_l = \left[ v_{\perp,l+1}, v_{\perp,l-1}, \sqrt{2} v_{\parallel,l} \right] \). The integral over \( v_{\parallel} \), appearing in the energy absorption kernel, is evaluated using the Plemelj relation.\(^4\) As an alternative way, the flux surface averaged power is calculated using bounce averaged coefficients that are derived directly with wave fields, expressed as

\[ P_{RF} = \frac{-4\pi e E_{\text{norm}}}{\phi} \int_{v_{\perp,0}}^{\infty} \int_{v_{\perp,0}}^{\infty} \int_{v_{\parallel,0}}^{\infty} \left( B_0 \frac{\partial f_0}{\partial v_0} + C_0 \frac{\partial f_0}{\partial \psi_0} \right) \). \quad (5)

Here, bounce averaged coefficients and other coefficients appearing in Eq. (5) are fully expressed in Ref. [4]. Good agreement between Eqs. (4) and (5) was validated in Ref. [4].

In Sec. III, we compare the absorbed ICRF wave powers computed from ORBIT-RF and AORSA in the limit of linear absorption, using the flux surface averaging of Eqs. (1) and (5) in ICRF wave heating experimental conditions in the Alcator C-Mod and DIII-D tokamaks.
III. RESULTS

A. “Model problem”

The codes ORBIT-RF and AORSA both read numerical equilibrium data in EQDSK format calculated by EFIT. Ignoring up-shift of wave spectrum, a single $k_{||}$, defined by $k_{||} = N_q / R$, is assumed where $R$ is the major radius and $N_q$ the toroidal mode number. Even though AORSA uses a hot plasma dispersion, it solves Maxwell’s equations in physical space, hence cannot decouple the $k_\perp$ for each plasma wave for a given $k_{||}$. Therefore, in ORBIT-RF, $k_\perp$, appearing in Eq. (2) as an argument of Bessel function, is calculated from cold plasma dispersion relation. Due to a similar reason, the $e^{2i\rho k}$ term in Eq. (2) is also assumed to be 1 by assuming $k_\perp = k_x$. Justification of this approximation will be discussed later.

Plasma spatial distribution is generated from an analytical fitting based on experimental data, expressed by $X(\rho) = X(\text{limiter}) + (X(0) - X(\text{limiter})) \times (1 - \rho^\beta_p)^\alpha_x$, where $X$ represents densities or temperatures, $\rho = \sqrt{\psi_p}$, $\psi_p$ the poloidal flux. The initial plasma distribution in velocity space is assumed to be Maxwellian. Since AORSA does not evolve its distribution function in time, for comparison, we impose the following constraints in ORBIT-RF: test ions are not updated in energy and pitch after RF “kicks”, and both random “kicks” and collisions are also turned off. To enhance Monte-Carlo statistics, one million particles are simulated. Initial pitches of test particles are uniformly distributed from -1 to 1 for an isotropic distribution in velocity space. Since AORSA calculates wave electric fields per unit antenna current, they are rescaled with experimental input power under the assumption of its complete absorption by the plasmas according to the linear theory.

B. Comparison of ICRF wave powers absorbed by the plasma

First, the C-Mod discharge #11051206002, thermal hydrogen minority fundamental harmonic heating experiment, is simulated. The 80 MHz ICRF wave is launched into a background deuterium (92%)-hydrogen (8%) plasma with ICRF power $P_{\text{EXP}} = 0.6$ MW. The toroidal magnetic field $B_0 = 5.4$ T, the plasma current $I_p = 0.8$ MA, the minor radius $a = 25$ cm and the magnetic axis $R \sim 70$ cm. Fundamental harmonic resonance is located near at magnetic axis. Plasma parameters are $n_e(0) = 12.8 \times 10^{19}$ m$^{-3}$, $n_e(\text{limiter}) = 3.42 \times 10^{19}$ m$^{-3}$, $\alpha_{ne} = 0.6$, $\beta_{ne} = 1.4$, $n_H(0) = 1.27 \times 10^{19}$ m$^{-3}$, $n_H(\text{limiter}) = 0.34 \times 10^{19}$ m$^{-3}$, $\alpha_{nH} = 0.6$, $\beta_{nH} = 1.4$, $T_e(0) = 2.6$ keV, $T_e(\text{limiter}) = 0.052$ keV, $\alpha_{Te}(0) = 1.3$, $\beta_{Te} = 1.9$, $T_D(0) = 2.0$ keV,
$T_D(\text{limiter}) = 0.052 \text{ keV}, \ \alpha_{TD} = 1.3 \text{ and } \beta_{TD} = 1.9$. Figure 1 shows wave electric fields ($E_+$ and $E_-$) computed from AORSA when $N_p = 10$ for 60 keV hydrogen Maxwellians. For 25 keV Maxwellian, similar pictures are shown, though not shown in this paper. In Fig. 2, the absorbed powers computed from ORBIT-RF (reds and magentas) coupled with AORSA wave fields (Fig. 1) are compared with those directly computed from AORSA (blues and greens). Quantitative agreements are found with $E_+$ only and also with total $E$ in both $T_H = 25$ keV and 60 keV cases. As expected, the $E_-$ contribution to absorbed power is minor at fundamental harmonic heating regime due to small $J_2$ in Eq. (2) and (5).

![Fig. 1. Wave electric fields computed from AORSA for (a) $E_+$ (left-hand polarized component) and (b) $E_-$ (right-hand polarized component) when $N_p = 10$ for 60 keV hydrogen Maxwellian in the Alcator C-Mod discharge #11051206002.](image1)

![Fig. 2. The ICRF wave power absorptions computed from ORBIT-RF (reds and magentas) coupled with AORSA wave fields and directly from AORSA (blues and greens) for (a) 25 keV and (b) 60 keV hydrogen Maxwellians in the C-Mod discharge #11051206002.](image2)

Secondly, the DIII-D discharge #122080, with neutral beam heating and high harmonic ICRF heating, is simulated. Injected deuterium beam ions are assumed to be a Maxwellian distribution, $R \sim 173 \text{ cm}, \ a = 60 \text{ cm}, B_0 = 1.87 \text{ T},$
$I_p = 1.2$ MA and $P_{\text{exp}} = 1.1$ MW. Plasma parameters are $n_e(0) = 6.74 \times 10^{19}$ m$^{-3}$, $n_e(\text{limiter}) = 1.86 \times 10^{19}$ m$^{-3}$, $\alpha_{n_e} = 0.5$, $\beta_{n_e} = 2.0$, $n_{\text{beam}}(0) = 2.85 \times 10^{18}$ m$^{-3}$, $n_{\text{beam}}(\text{limiter}) = 3.25 \times 10^{15}$ m$^{-3}$, $\alpha_{n_{\text{beam}}} = 0.5$, $\beta_{n_{\text{beam}}} = 2.0$, $T_e(0) = 1.92$ keV, $T_e(\text{limiter}) = 0.03$ keV, $\alpha_{T_e} = 2.0$, $\beta_{T_e} = 2.0$, $T_D(0) = 2.21$ keV, $T_D(\text{limiter}) = 0.19$ keV, $\alpha_{T_D} = 2.0$ and $\beta_{T_D} = 2.0$. Simulations are separately preformed with each of two ICRF wave frequencies: 60 MHz and 116 MHz. The 60 MHz wave interacts resonantly with beam ions at 4th ($R \sim 164$ cm) and 5th ($R \sim 204$ cm) harmonics, while the 116 MHz wave at 8th ($R \sim 171$ cm), 9th ($R \sim 191$ cm) and 10th ($R \sim 210$ cm). Since 5th and 10th resonance layers are located close to the wall ($R_{\text{wall}} \sim 220$ cm), the presence of energetic particles near the wall may lead to their prompt loss due to interception of their large banana orbits by the first wall before they transit the resonant layers, which can significantly modify the plasma distribution and the ICRF wave power absorption near the wall.

As mentioned in Sec. I, ORBIT-RF can study this effect due to its numerical approach following an orbit trajectory of each single particle. Therefore, to investigate loss effect of energetic particles on the ICRF power absorption due to its large orbit width, we perform ORBIT-RF and AORSA simulations when Maxwellian beam temperature profile is taken to be uniform spatially. Figures 3 and 4 show wave electric fields $E_+$ (V/m) and $E_-$ (V/m) computed from AORSA for 60 MHz and 116 MHZ ICRF waves in 60 keV beam Maxwellian when $N_{\psi} = 13$, showing $E_-$ is comparable to $E_+$ in magnitudes. Similar wave fields are also computed for the 25 keV beam Maxwellian. In Fig. 5, $k_\perp$ (m$^{-1}$) computed from cold plasma dispersion relation is plotted for 60 MHz and 116 MHz, which are implemented in ORBIT-RF to calculate Bessel functions in Eq. (2).

Fig. 3. Wave electric fields computed from AORSA for (a) $E_+$ and (b) $E_-$ when $N_{\psi} = 13$ for 25 keV beam Maxwellian in the DIII-D discharge #122080.
In Figs. 6 and 7, ICRF wave powers absorbed by beam ions are compared for two beam MaxweIl& 133 lians temperatures (25 keV and 60 keV) with 60 MHz and 116 MHz frequencies. Qualitative agreements are found within a factor of two when absorbed powers computed with total $E$ are compared, whereas, comparisons with only $E_+$ indicate more quantitative agreements. Similar agreements are also found in the comparisons with only $E_-$, though not shown in this paper. This suggests that the difference in the comparisons with total $E$ is due to of the uncertainty in assuming $e^{2\pi ik} = 1$ in Eq. (2), rather than from the cold plasma approximation for $k_\perp$ in ORBIT-
RF. These approximations were found to be inconsequential in the C-Mod fundamental harmonic heating simulations due to the negligible contribution of $E_\perp$ to absorbed power due to small $k_\perp\rho$ in Eqs. (2) and (5). However, large $k_\perp\rho$ in the DIII-D high harmonic heating regimes produces significant contribution from $E_\perp$ to absorbed power. To remove this uncertainty, in the future we shall investigate an alternative numerical algorithm, passing directly the Q-L diffusion coefficients computed from AORSA on to ORBIT-RF.

![Graph](image1)

Fig. 6. The 60 MHz ICRF wave powers absorbed by (a) 25 keV Maxwellian beam ions and (b) 60 keV Maxwellian beam ions with no radial beam temperature profile in the DIII-D discharge #122080.

![Graph](image2)

Fig. 7. The 116 MHz ICRF wave powers absorbed by (a) 25 keV Maxwellian beam ions and (b) 60 keV Maxwellian beam ions with no radial beam temperature profile in the DIII-D discharge #122080.

Large differences between AORSA and ORBIT-RF are also indicated in Figs. 6 and 7 in regions of $\sqrt{\psi_p} > 0.6$, which can be explained by orbit loss mechanism of energetic particles to the wall. Figure 8 shows that many of energetic beam ions, located initially near the wall due to spatially constant beam temperature used in the modeling, are lost to the wall before they interact with ICRF wave at resonance layers due to its large banana orbit width. It is shown that more particles are lost in the 60 keV case than in the 25 keV case, consistent with this picture. Since AORSA currently does not take into account the
loss effect of the energetic particles on the distribution function, AORSA computes higher power absorption near the wall than that from ORBIT-RF, as shown in Figs. 6 and 7. Unrelated to this, in the 116 MHz wave — 60 keV beam Maxwellian simulation at high harmonics [Fig. 7(b)], ORBIT-RF computes more near-axis power absorption than that from AORSA.

Fig. 8. Energetic beam ions lost to the wall due to their large banana orbit widths for 25 keV and 60 keV Maxwellian beam ions in 116 MHz DIII-D discharge #122080.

For better understanding of the finite orbit mechanism, similar simulations are performed using more realistic spatial beam temperature profile with $\alpha_{T,\text{beam}} = 2.0$ and $\beta_{T,\text{beam}} = 2.0$. In Figs. 9 and 10, absorbed powers computed with only $E_\parallel$ are compared, showing better agreements in all cases due to no loss of energetic particles near the wall. As shown in Fig. 10(b), ORBIT-RF still retains the feature of having more absorbed power near to the magnetic axis than that from AORSA [as shown similarly in Fig. 7(b)]. A picture emerges from further analysis. Figure 11 shows corresponding contour plots of absorbed powers in $(R, Z)$ coordinate, indicating that at the 8th resonance layer, peak location of absorbed power from ORBIT-RF is slightly lower in a vertical $Z$ direction than that from AORSA. Figure 12 shows an orbit of 206 keV beam ion that is selected from modeled 60 keV beam Maxwellian distribution, starting closely at position 1 and passes through 8th and 9th resonance locations, consecutively. Due to its large banana orbit width, this particle interacts with the ICRF wave at four resonant points, marked as 1, 2, 3, and 4. The “kicks” computed from ORBIT-RF at 1 and 4 contribute most to the peak in the absorbed power at $\sqrt{\psi_R} = 0.1$ in Fig. 10(b). Figure 13 shows resonant interaction times computed using Eq. (5) in Sec. II with orbit topology, explaining they are much longer at the inner resonant locations (1 and 4) than outer locations (2 and 3). This effect, which only manifests itself with finite orbits, provides a credible explanation why more power absorption occurs near the magnetic axis for ORBIT-RF.
Fig. 9. The 60 MHz ICRF wave powers absorbed by (a) 25 keV Maxwellian beam ions and (b) 60 keV Maxwellian beam ions with radial beam temperature profile in the DIII-D discharge #122080.

Fig. 10. The 116 MHz ICRF wave powers absorbed by (a) 25 keV Maxwellian beam ions and (b) 60 keV Maxwellian beam ions with radial beam temperature profile in the DIII-D discharge #122080.

Fig. 11. Contour plots of absorbed ICRF wave powers in $(R, Z)$ coordinate from AORSA and ORBIT-RF.
Fig. 12. Large banana orbit of 206 keV beam ion that is selected from modeled 60 keV beam Maxwellian distribution in 116 MHz case.

Fig. 13. Resonant interaction times computed with orbit topology at four resonant locations from ORBIT-RF.
IV. SUMMARY

As a first step in coupling the 2-D full wave solver AORSA and the 5-D Monte-Carlo plasma distribution solver ORBIT-RF, the ICRF wave power absorption predicted by ORBIT-RF using wave fields from AORSA computed with Maxwellian distributions has been verified against linear absorption directly evaluated by AORSA. For comparison, we set up “model problems” that enforce some simplifications in plasma and wave conditions, as applied to the ICRF wave heating experimental conditions over various ion cyclotron harmonics in the Alcator C-Mod and DIII-D tokamaks. The purpose is to test the validity of the Monte-Carlo particle approach to model ICRF heating and to understand finite orbit effects of energetic particles on the ICRF wave power absorption that cannot be addressed from other approaches that assume zero orbit width.

For the C-Mod minority fundamental harmonic heating, ORBIT-RF reproduces linear power absorption computed from AORSA. Quantitative agreements are found not only in results with $E_+^+$ only term but also those with total $E$. However, in the DIII-D beam ion high harmonic heating, absorbed powers computed with total $E$ indicate qualitative agreements within a factor of two, while comparisons with only $E_+$ or $E_-$ show better agreements, which suggest the assumption of $k_\parallel = k_\parallel$ in “model problem” is not valid in the DIII-D beam ion high harmonic heating regime due to a significant contribution from $E_-$ to absorbed power due to large $k_\parallel$. To remove this approximation, an alternative numerical algorithm, passing directly the Q-L diffusion coefficients computed from AORSA on to ORBIT-RF, is being considered.

Comparisons of absorber powers computed with only $E_+$ show better agreements. Important finite orbit effects of energetic particles on ICRF wave power absorption emerge from the comparison. When resonant layers are located near the wall, the presence of energetic particles near the wall can lead to prompt loss to the wall at high edge density due to their large banana widths, and consequently ORBIT-RF computes less power absorption near the wall than AORSA. More interestingly, in the 116MH wave — 60 keV beam Maxwellian simulation at high harmonics (8th, 9th and 10th), ORBIT-RF computes more near-axis power absorption than AORSA. This is understood as due to two effects from ORBIT-RF. As a first effect, energetic trapped particles pass through four resonant points due to its large banana orbit width, which results in more near-axis power absorption, while AORSA assumes always two resonance points with zero orbit approximation. Secondly, the wave-particle interaction time is more accurately computed with orbit topology in ORBIT-RF. An analysis of a single banana orbit with four resonant points shows the interaction time to be much longer at inner resonant points.
than the outer, which also play a role in generating more power absorption near to magnetic axis. Our result verifies the validity of the Monte-Carlo particle approach for all ICRF harmonics heating regimes and the importance of finite orbit effects of trapped particles in theoretical modeling of high harmonic ICRF wave — nonthermal plasma interactions. It supports proceeding to the next phase of coupling between ORBIT-RF and AORSA to study non-Maxwellian modifications to ICRF power absorption.
REFERENCES


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