Measurement of Resistive Wall Mode Stability in Rotating High Beta Plasmas


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20th IAEA Fusion Energy Conference, 1 - 6 November 2004, Vilamoura, Portugal 2004

Stabilization of the Resistive Wall Mode (RWM) can extend the operating regime from the no-wall up to the ideal wall limit

- Operation above the no-wall limit particularly important for advanced tokamak (AT) scenarios
  - ATs rely on a large fraction of bootstrap current
  - Broad current profiles greatly benefit from wall stabilization

- Operation in the wall stabilized regime with \( \beta_N \sim 6 \) and \( \beta_T \) reaching 6%
Outline

- Passive measurement of the toroidal plasma rotation required for stability $\Omega_{\text{crit}}$
  - $\Omega_{\text{crit}}$ in two scenarios (low-$l_i$ and moderate-$l_i$)
  - Comparison with MARS calculations
- Active measurement of RWM growth rate $\gamma_{\text{RWM}}$ and mode rotation frequency $\omega_{\text{RWM}}$
  - Static and dynamic response to externally applied pulsed n=1 fields yields $\gamma_{\text{RWM}}$ and $\omega_{\text{RWM}}$
  - Measurement of $\gamma_{\text{RWM}}$ and $\omega_{\text{RWM}}$ with pulsed fields in the low-$l_i$ scenario
  - Plasma response to externally applied rotating n=1 fields yields $\gamma_{\text{RWM}}$ and $\omega_{\text{RWM}}$ (MHD spectroscopy)
  - Measurement of $\gamma_{\text{RWM}}$ and $\omega_{\text{RWM}}$ with rotating fields in the moderate-$l_i$ scenario
  - Comparison with MARS calculations
- Application of MHD spectroscopy as a continuous stability measurement
- MARS predictions of $\Omega_{\text{crit}}$ for DIII-D AT scenarios

Resistive Wall mode (RWM):
Free-boundary ideal MHD kink mode in the presence of a resistive wall
- Observed between no-wall and ideal wall ideal MHD limit
- “Slow” RWM growth $\gamma_{\text{RWM}} \sim T_w^{-1}$
  → Stabilization by feedback control
- “Slow” mode rotation $\omega_{\text{RWM}} \ll \Omega_{\text{rot}}$
  → Quasi-static magnetic perturbation in a fast plasma flow

Plasma rotation predicted to stabilize the RWM

- Understanding of dissipative process essential for extrapolation to future experiments
Several dissipation models are proposed

- **Sound wave damping**: perturbed plasma rotation $v_1$ couples to sound waves, which are subject to ion Landau damping [Bondeson and Ward, *Phys Rev Lett* 72 (1994) 2709]
  - Described by a parallel viscous force: $F_{\text{visc}} = -\kappa_{ii} |k_i| v_{th,i} \rho v_{th}$
  - Cylindrical theory with a free parameter $\kappa_{ii}$ to describe the effects of toroidicty and shaping
- **Kinetic damping**: electromagnetic perturbation kinetically damped through Landau damping process [Bondeson and Chu, *Phys Plasmas* 3 (1996) 3013]
  - No adjustable parameter
- **Additional stabilization models**
- **Main computational tool is the MARS-F code** [Liu et al, *Phys Plasmas* 7 (2000) 3681], which includes the sound wave or kinetic damping model

How much plasma rotation is required to stabilize the $n=1$ RWM?

- Passive measurement of $\Omega_{\text{crit}}$
  - Insufficient error field correction causes slow-down of toroidal rotation
  - Onset of RWM marks $\Omega_{\text{crit}}$
- Systematic scan of $\beta$ in a low-$\ell$ plasma [R.J. La Haye et al, accepted for publication in *Nucl. Fusion*]
  - $\Omega_{\text{crit}}$ scales with $\tau_A^{-1}$
- Additional data in a moderate-$\ell$ plasma
Equilibrium profiles of low-$I_i$ and moderate-$I_i$ scenarios

• Low-$I_i$ scenario greatly benefits from wall stabilization
  - $\beta_{N,\text{no-wall}} \sim 1.6 \sim 2.4 I_i$
  - $\beta_{N,\text{ideal-wall}} \sim 3.2 (\sim 4.8 I_i)$

• Moderate-$I_i$ scenario has a higher no-wall limit
  - $\beta_{N,\text{no-wall}} \sim 2.0 \sim 2.4 I_i$
  - $\beta_{N,\text{ideal-wall}} \sim 3.2 (\sim 3.8 I_i)$

• Moderate-$I_i$ scenario has a higher safety factor $q_{95}$ (includes $q=5$ and 6 surfaces)

MARS predictions of $\Omega_{\text{crit}}$ in qualitative agreement with measurements

• Low-$I_i$ scenario yields $\Omega_{\text{crit}} \tau_A \sim 0.02$ with weak $\beta$ dependence

• Both damping models predict $\Omega_{\text{crit}}$ within a factor of 2
  - Kinetic damping generally underestimates $\Omega_{\text{crit}}$

• Both models predict the trend of a lower $\Omega_{\text{crit}}$ in the moderate-$I_i$ scenario
Probe RWM stability by applying an external resonant magnetic field while the plasma remains stable

- **Active MHD spectroscopy**
  Drive a low amplitude perturbation at various frequencies using external antennas and extract the plasma response with synchronous detection.


- **Resonant field amplification (RFA):**
  - Source of external field can be currents in control coils or intrinsic error field
  - RFA amplitude defined as ratio of plasma response and applied field
    \[ A_{RFA,s} = \frac{B_s - B_{s,ext}}{B_{s,ext}} \]

  Complex notation: \( f(t, \varphi) = \Re( F(t) \cdot e^{-i\varphi} ) \) where \( \varphi \) is the toroidal angle

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**DIII-D has versatile sets of antennas and detectors**

**Experimental setup:**

**Antennas:**
- 6 external (C-coil) and 12 internal (I-coil) saddle coils

  - Static or rotating magnetic field with large overlap with RWM structure at the wall.

**Detectors:**
- Toroidal arrays of saddle loops and poloidal field probes
Plasma response is in the linear regime

- Applied I-coil field $\sim 10$ Gauss/kA
- Linear response
  - Amplitude depends on sensor:
    - "MPI": midplane poloidal field probes
    - "ESL": midplane saddle loops

Single-mode models describe interaction between externally applied fields and the RWM


\[ \frac{d B_s}{dt} - \tau_w \frac{d}{dt}(B_s) = M_{sc}^{\prime} I_c \]

for the perturbed field $B_s$ and currents in the control coils $I_c$

- The RWM growth rate for in the absence of external currents $\gamma_0 = \gamma_{RWM} + i \omega_{RWM}$ is given by the dispersion relation:
  - ‘Simple’ RWM model:
    \[ \frac{\Lambda}{K} \]
  - Extended lumped parameter model:
    \[ \frac{\delta W_{no-wall} + i \Omega_{rot} D}{\delta W_{ideal-wall} + i \Omega_{rot} D} \]
  - Ideal MHD with rotation and dissipation:
    \[ \frac{\Lambda}{K} \]

from MARS
Dynamic response to resonant field pulses consistent with single-mode model

- Response to static pulse
  \[ A_{RFA,s} = c_s \frac{1 + \gamma_0 \tau_W}{-\gamma_0 \tau_W} \]
  with \( c_s = M_s^* / M_s \) being the ratio of the resonant component and the total externally applied field yields \( \gamma_0 \)

- Decay of perturbation after pulse \( B_s(t) = B_s(t_0) e^{\gamma_0 t} \) yields independent measurement of \( \gamma_0 \)


Dynamic response to C-coil pulses yields a measurement of the RWM damping rate and mode rotation frequency

- Low-\( l \) target (\( l \sim 0.67 \))
- Optimum error field correction sustains plasma rotation at \( \Omega_{\text{rot}} \tau_W \sim 0.02 \) at \( q = 2 \)
- Apply \( n=1 \) field pulses with C-coil
- Best fit of RFA amplitude, phase and exponential decay to single-mode model yields \( \gamma_0 \)
- Plasma approaches marginal stability at \( C_\beta \sim 0.6 \)
  - consistent with measured \( \Omega_{\text{crit}} \tau_W \sim 0.02 \)
- Mode rotation frequency is low (fraction of \( \tau_W^{-1} \)) and has a weak \( \beta \) dependence

MHD spectroscopy probes the RWM stability while the plasma remains stable

- **Moderate-\( l \) target (\( l \approx 0.85 \))
- Apply rotating \( n = 1 \) field with I-coil
- Coherent detection

- Largest plasma response for slowly co-rotating field
- Plasma response leads external field if rotation slower and trails if rotation faster than rotation of largest response

RFA peaks when the externally applied field rotates with the mode rotation frequency


\[
A_{RFA,s} = c_s \cdot \frac{1 + \gamma_0 \tau_w}{i \omega_{ext} \tau_w - \gamma_0 \tau_w}
\]

- Fit of \( \gamma_0 \) and \( c_s \) results in good agreement
  - Single-mode model applicable
  - RFA spectrum yields a measurement of \( \gamma_0 \) (**MHD spectroscopy**)
MHD spectroscopy yields a measurement of the RWM damping rate and mode rotation frequency

- MHD spectroscopy in moderate-$l$ target yields $\beta$ dependence of $\gamma_0$
- Optimum error field correction sustains plasma rotation at $\Omega_{\text{rot}} \tau_W \sim 0.02$ at $q=2$

- Growth rate is lower than in low-$l$ scenario and remains below marginal stability up to $C_\beta \sim 1$
  - consistent with measured $\Omega_{\text{crit}} \sim 0.01 \tau_W^{-1} \ll \Omega_{\text{rot}}$
- Mode rotation frequency is low (fraction of $\tau_W^{-1}$) and has a weak $\beta$ dependence, similar to low-$l$ scenario

Comparison with MARS

- Both models predict $\gamma_{\text{RWM}}$ too low
- Kinetic damping predicts experimental $\omega_{\text{RWM}}$ while the sound wave damping prediction is too high
Active RWM spectroscopy yields a continuous “non-perturbative” measurement of the stability

- With $c_s$ known, $A_{RFAS}$ becomes a continuous measurement of $\gamma_0$.

\[ \gamma_0 = \frac{i\omega_{ext} A_{RFAS}}{c_s} / \frac{A_{RFAS}}{c_s} + 1 \]

- Potential for real-time indication of the approach to the stability limit

MARS predicts significant contribution of the $q=2$ surface towards rotational stabilization

- Set of DIII-D advanced tokamak equilibria with various $q_{\text{min}}$ and $q_a=7.2$
- Calculate $\Omega_{\text{crit}}$ for stability up to ideal wall limit
- Sound wave damping predicts a weak increase of $\Omega_{\text{crit}}$ for $q_{\text{min}} > 3/2$ and a strong increase of $\Omega_{\text{crit}}$ for $q_{\text{min}} > 2$
- Kinetic damping predicts a significant increase of $\Omega_{\text{crit}}$ for $q_{\text{min}} > 2$

$\rightarrow$ DIII-D AT scenarios could require direct RWM feedback control
Summary

- Interaction between an externally applied magnetic field and a high-$\beta$ plasma at various frequencies is well described by a single mode approach
  - Validation of the single mode approach (basis of RWM feedback models)
  - Absolute measurement of RWM damping rate $\gamma_{RWM}$ and mode rotation frequency $\omega_{RWM}$
- Passive measurement of the critical plasma rotation $\Omega_{crit}$, and active measurement of $\gamma_{RWM}$ and $\omega_{RWM}$ carried out in two scenarios (low-$l$ and moderate-$l$)
  - Low-$l$ scenario requires more rotation for stability → importance of rational surfaces at plasma edge for damping process
- Comparison of RWM stability measurements with sound wave damping and kinetic damping implemented in the MARS code
  - Both damping models reproduce the weaker damping in the low-$l$ scenario and predict $\Omega_{crit}$ within factor of 2
  - Both damping models overestimate $|\gamma_{RWM}|$ or $|\omega_{RWM}|$ or both
- Progress towards a quantitative test of our understanding of rotational stabilization requires further development of experiment and theory

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