Sawtooth Oscillations in Shaped Plasmas

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Trying to alter the magnetic well in the sawtooth region results in major changes in plasma behavior.

• Sawtooth behavior is altered
• Ion energy transport is altered.
• Particle transport is altered.
• Electron energy transport is radically altered
  ▪ parallel resistivity \((T_e^{3/2})\) is altered
  ▪ \(q\) profile evolves differently
  ▪ \(n=1\) stability is altered

The bean collapse is an internal kink mode.
The oval collapse is a quasi-interchange mode.

What is not altered is the resistive or ideal interchange criterion.

Over a large central region of the plasma \((\rho < r_i)\) the interchange criterion is virtually always violated.

Nevertheless, I will try to convince you of the importance of interchange stability.
The Idea of the Experiment
Interchange Stability in Bean and Oval Shapes

Shaping can separate interchange from internal kink stability ($q=1$).
Differences! The Sawteeth - In Beans and Ovals

- Twice the Amplitude
  - $\delta T_e / T_e \approx 3/4$
  - $\delta n_e / n_e \approx 1/5$
- Twice the Period
- Violent event
- Successor $m/n=1/1$ Oscillations
  - $\delta T_e / T_e \approx 1/3$
  - $\delta n_e / n_e \approx 0$
- Mild Event
- Precursor $m/n=1/1$ Oscillations
The collapse: an ECE color map shows the qualitative differences in the collapse.

No visible precursor to the collapse. Collapse is followed by formation of an island structure that exists for a short period after collapse. Collapse time \( \sim 40 \mu s \)

Precursor + Fishbone =>
Saturated State => Mixing =>
Increased Modulation Depth =>
Will collapse in a few cycles =>
Near axisymmetric State.
Collapse time \( \sim 70 \mu s \)
Examine the FFT of the cross correlation of ECE signals with magnetic n=1 signal.

(No coherent structure before collapse.)
Double island structure after collapse (inboard & outboard 180° phase jumps).

Single inboard 180° phase jump before collapse is the signature of quasi-interchange.
FFT Model
Inboard Phase Jump Indicates Quasi-Interchange

Components of model:
n=1 eigenfunction and a temperature profile

Fluctuation signal will be
\[ S = \xi(r,\theta,t) \cdot T_e'(r,t) \]

\[ \xi(r,\theta,t) = \xi_1(r)\cos[\theta-t] + \xi_2(r)\cos[2\cdot\theta-t] \]

Phase jump from Cos terms and jump back is from Sin terms. Separation depends on slope of the m=1 component as it intersects the m=2 component of \( \xi \). If the m=1 component is kink-like (top hat) the terms cancel and the phase jump does not occur.
Beans develop monotonic q profiles, $q_0 < 0.9$

Double $q=1$ crossing like ECE correlation at $t_c + 13$ ms

Does not collapse at minimum $q_0$ and maximum $p$. 
Ovals develop little central shear, $q_0 > 0.95$

Higher current ovals show $q_0 < 1$ and a larger pressure change than the case shown here but retain single inboard phase jump in ECE.
Stability analysis consistent with ECE FFT’s

Calculations of the eigenmodes for experimental equilibria are consistent with the expected distinction between QI and RIK

![Graph showing eigenmodes](image)

- **Oval:** $q_0 = 0.95$
- **Bean:** $q_0 = 0.85$

$\text{Mode Amplitude (au)}$ vs. $\text{Normalized Minor Radius}$

- $m=1$
- $m=2$

$\leftarrow q=1$ locations
Characteristic Behaviors during the Sawtooth Ramp

Central $T_e$, $T_i$, and internal $B_\theta$:

• $T_i > T_e$

Bean:
• $T_i \approx T_e$
• $T_i$, $v_\phi$ roll over while $T_e$ is still increasing.
• $B_\theta(t)$ crashes

Oval:
• $T_i > T_e$
• $T_i$ does not roll over (relative to $T_e$)
• $B_\theta(t)$ meanders

$\dagger B_\theta$ from MSE at $R=1.62$ m, $\rho \approx 0.12$
Impulse Response to ECH at $\rho \approx 0.05$

Electrons respond locally in beans.

Electrons do not respond locally in ovals.

After $m/n=1/1(\bullet)$ is present, a weak electron response
Central Thermal Diffusivities
(from TRANSP analysis)

\( \chi \) – volume-averaged: axis to \( \rho = 0.3 \) vs normalized time

\[ m^2/s \]

\[ Time / \tau_s \]
Central Thermal Diffusivities
(from TRANSP analysis)

\( \chi \) – volume-averaged: axis to \( \rho = 0.3 \) vs normalized time

\( \chi_i \)
\( \chi_e \)
\( \chi_i^{NC} \)

\( \chi_i \approx \chi_i^{NC} \)
Central Thermal Diffusivities
(from TRANSP analysis)

\( \chi \) – volume-averaged: axis to \( \rho = 0.3 \) vs normalized time

\( \chi_i \approx \chi_i^{NC} \)

\( \chi_e \approx \chi_e^{PC} \)
Quasi-Interchange and Resistive Internal Kink

• In both bean and oval shapes, we can redo our TRANSP runs, using neoclassical resistivity rather than the measured $q$ profiles, and then compare the calculated pitch angle ($\gamma_P = \tan^{-1}\{B_z/B_\phi\}$) to the measured values. The difference is well within the 0.3 deg. statistical error in $\gamma_P$.

• We have no direct evidence that the bean and oval shapes “cause” RIK and QI respectively. Rather, we have evidence the shapes result in changes in $T_e$ leading to differing current profiles.

• The shapes result in drastically different electron confinement, resulting in different resistivity profiles evolving different $q$ profiles and leading to the different sawtooth behavior. 
• It is not clear whether shape, *per se*, contributes to the difference in the collapse.
Interchange Stability Fails

• The resistive interchange criterion ($D_R$) is positive (unstable) over the region inside $r_i$ over most of the sawtooth period in both bean and oval plasmas.

• $D_R$ can be separated into magnetic well, shear and $\nabla p$ terms. In these experiments the latter 2 terms are negligible. The problem is the lies with the (lack of) well.

• Said differently, the normal curvature, $\kappa_n=(\partial / \partial \psi <B^2+2\mu_0 p>/B^2$, is near 0 inside $r_i$ in both oval and bean. When we make the bean (as compared to the oval) we do increase $(\partial / \partial \psi <B^2>/B^2$, but then the plasma raises $\partial p/\partial \psi$ and $\kappa_n$ is unchanged from the oval. (Also, $\partial / \partial \psi <B^2>$ is reduced a bit as $q_0$ drops.)
Interchange Stability

The interchange stability criterion is routinely violated -- but we observe major differences in plasma behavior and transport. At the global level we see the change between QI and RIK. 

The experimental evidence is that changes in $\frac{\partial}{\partial \psi} \frac{\langle B^2 \rangle}{B^2}$ cannot be ignored. I think this demonstrates a conceptual validity for interchange stability, but the calculation of the stability criterion fails. 

• One possibility is that kinetic corrections† are be needed. These might remove the violation of the interchange criterion, but electron transport appears to predominate and is not explained by interchange stability in any straightforward manner.


We note that, were these results generally applicable to toroidal device, the effects would be minimized in a hot-ion mode and be more severe in an $\alpha$-heated device than shown here, where $P_{be} \approx P_{bi}$. 